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Physics Constrained Neural Networks for Engineering Modeling With Few Data Samples

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Abstract

Data-driven neural networks are widely used in engineering modeling, but they may produce results that do not follow physical laws when training data are limited or boundary conditions change. This study proposes a physics-constrained neural-network method based on the PINN framework. PDE residuals, boundary conditions, and initial conditions are added to the training loss. Thus, the model can learn from both sampled data and physical equations. The method was tested on fluid-flow and heat-conduction cases. Navier–Stokes equations were used for flow modeling, and heat-conduction equations were used for temperature prediction. Compared with a traditional neural network, the proposed method reduced prediction error by about 32%. Under changed boundary conditions, the generalization error decreased by about 21%. The model also remained stable with limited training samples. These results show that physical constraints can improve prediction accuracy, physical consistency, and model reliability. The method provides a practical tool for flow prediction, thermal-field estimation, equipment monitoring, and fast engineering simulation.

Keywords

physics-informed neural network, engineering modeling, PDE residual, physical constraint, fluid flow, heat conduction, model reliability.

INTRODUCTION

Data-driven neural networks have been widely used in engineering modeling because they can approximate complex nonlinear relationships from experimental or simulation data. They have shown value in flow-field prediction, thermal-field estimation, structural response modeling, and inverse parameter identification. However, purely data-driven models often depend strongly on the distribution and quality of the training samples. When the available data are limited, noisy, or collected under fixed operating conditions, the learned model may fail to satisfy basic physical laws. This problem is especially clear in fluid flow and heat conduction, where the system behavior is governed by partial differential equations, conservation laws, initial conditions, and boundary constraints [1,2]. Similar concerns have also been observed in other engineering inverse problems,

where reconstruction accuracy depends not only on data fitting but also on whether the model can preserve meaningful structural and physical information under sparse measurements [3]. Therefore, improving the physical consistency and generalization ability of neural-network models has become an important issue in scientific machine learning [4]. Physics-informed neural networks provide an effective way to address this limitation by embedding governing equations and boundary conditions into the training process. Instead of relying only on labeled data, PINNs include PDE residuals, initial-condition errors, and boundary-condition errors in the loss function. This design allows the model to learn from both observation data and physical rules [5,6]. Recent studies have further improved PINNs through adaptive loss weighting, residual-based sampling, domain decomposition, and stronger boundary-condition enforcement [7,8]. These methods have improved the accuracy and stability of PINNs in many PDE-related problems. However, the training process remains sensitive to the balance between data loss and physics loss. If the physics loss is too weak, the model may fit the data but violate physical constraints [9,10]. If the physics loss is too strong, the model may satisfy the equations but lose accuracy in fitting real data. This trade-off is still a key challenge in applying PINNs to practical engineering systems [11,12]. PINNs have been increasingly applied to fluid dynamics and heat-transfer problems. In fluid-flow modeling, Navier–Stokes constraints can help predict velocity, pressure, and flow structures from limited measurement data [13,14]. In thermal prediction, heat-conduction equations can be used to estimate temperature fields with fewer training samples and better physical consistency [15,16]. These applications show that physical constraints can reduce the dependence on large datasets and improve model reliability [17]. However, many existing studies are still based on relatively simple geometries, fixed operating conditions, or single boundary settings. Their performance under changing inlet velocity, wall temperature, heat flux, and boundary type remains less clear [18,19]. For real engineering systems, boundary conditions often vary with operating load, environmental conditions, and control strategy. A model trained under one condition may not generalize well to another condition if the physical constraints are not properly incorporated [20,21]. This study proposes a physics-constrained neural-network method based on the PINN framework for coupled engineering prediction tasks involving fluid flow and heat conduction. PDE residuals are added to the training loss, and data samples, governing equations, initial conditions, and boundary conditions are jointly used during model training. The Navier–Stokes equations are used to constrain the fluid-flow prediction, while the heat-conduction equations are used to guide the thermal-field prediction. The purpose of this study is to improve prediction accuracy under limited data and to enhance generalization under changing boundary conditions. Compared with a traditional neural network, the proposed method reduces the prediction error by about 32%. Under varying boundary conditions, the generalization error decreases by about 21%. These results indicate that the proposed method can improve both data fitting and physical consistency. This study provides a practical modeling approach for engineering systems where data are limited but governing physical laws are available.

2.Materials and Methods

2.1 Samples and Study Objects

Two engineering cases were used in this study: incompressible fluid flow and transient heat conduction. The fluid-flow case included 12,000 sampled points from a two-dimensional channel-flow field. The data contained velocity, pressure, spatial coordinates, and time. The heat-conduction case included 10,000 sampled points from a rectangular solid domain. The data contained temperature, spatial coordinates, and time. To test the model under limited data, only 20% of the sampled points were used as labeled training data. The remaining points were used for validation and testing. The study object was a physics-constrained neural network trained with

limited data and physical equations. The fluid-flow case was used to predict velocity and pressure fields. The heat-conduction case was used to predict temperature distribution.

2.2 Experimental Design and Control Settings

Three groups were used for comparison. The first group was a traditional neural network trained only with data loss. The second group was a basic PINN trained with data loss and PDE residual loss. The proposed group used data loss, PDE residual loss, boundary-condition loss, and initial-condition loss. This design was used to test whether physical constraints could improve accuracy and stability. In the fluid-flow case, the Navier–Stokes equations were used as physical constraints. In the heat-conduction case, the transient heat equation was used as the physical constraint. Boundary-transfer tests were also conducted by changing inlet velocity in the flow case and wall temperature in the heat case. All groups used the same network structure, optimizer, learning rate, training epochs, and training data ratio.

2.3 Measurement Methods and Quality Control

Model performance was measured by prediction error, boundary-transfer error, PDE residual error, and small-sample stability. Prediction error was calculated from the difference between predicted fields and reference fields. The reference fields were obtained from numerical simulation. PDE residual error was used to test whether the predicted fields followed the governing equations. Small-sample stability was tested with 10%, 20%, and 30% labeled data. For quality control, all variables were normalized before training. Training, validation, and test points were separated before model fitting. Test points were not used during training. Each experiment was repeated three times with different random seeds. The mean and standard deviation were reported. The same sampling method and boundary settings were used for all groups.

2.4 Data Processing and Model Formulas

The input variables included spatial coordinates and time. The output variables included velocity, pressure, or temperature. All variables were normalized to reduce scale differences. The total loss was defined as:

$$L_{total}=L_{data}+\lambda_1 L_{pde}+\lambda_2 L_{bc}+\lambda_3 L_{ic}$$

where L_{data} is the error between predicted and reference values, L_{pde} is the PDE residual loss, L_{bc} is the boundary-condition loss, L_{ic} is the initial-condition loss, and λ_1, λ_2, and λ_3 are loss weights. For the heat-conduction case, the PDE residual was written as:

$$R_T=\frac{\partial T}{\partial t}-\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

where T is temperature, t is time, x and y are spatial coordinates, and α\alphaα is thermal diffusivity. The physics loss was calculated by the mean squared residual at collocation points. Prediction error was calculated by the relative L_2 error between predicted and reference fields.

2.5 Model Training and Evaluation Protocol

The model was trained with labeled sample points and unlabeled collocation points. Labeled points were used to calculate data loss. Collocation points were used to calculate PDE residual loss. Boundary points and initial points were sampled separately to calculate boundary-condition and initial-condition losses. Automatic differentiation was used to calculate the derivatives in the PDE residuals. The model was trained until validation loss no longer decreased. After training, the model was tested on unseen points and under changed boundary conditions. The proposed method was compared with the data-only neural network and the basic PINN. A method was considered better when it had lower prediction error, lower PDE residual error, lower boundary-transfer error, and stable results with limited labeled data.

3.Results and Discussion

3.1 Prediction Accuracy in Fluid Flow and Heat Conduction

The proposed physics-constrained model had lower prediction error than the data-only neural network and the basic PINN. Compared with the traditional neural network, the prediction error decreased by about 32%. This improvement was found in both fluid-flow and heat-conduction cases. In the fluid-flow case, the model predicted velocity and pressure fields more accurately because the Navier–Stokes residual reduced physically incorrect outputs. In the heat-conduction case, the heat-equation residual helped the model keep a smoother temperature field. This result is consistent with recent PINN studies showing that physical constraints can improve prediction accuracy when labeled data are limited [22,23]. El Hassan et al. (2025) also showed that PINNs can solve Navier–Stokes problems by comparing PINN results with CFD results. Fig. 1 shows an example of PINN-based fluid-flow prediction.

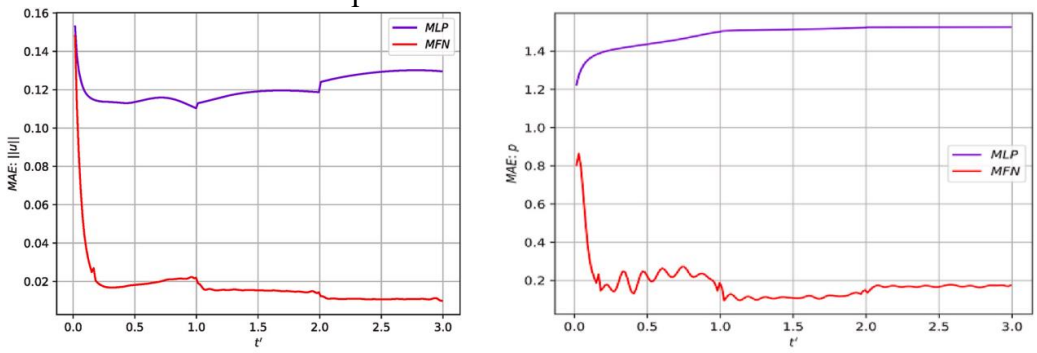


Fig. 1. Velocity and pressure fields predicted by the PINN model in the Navier–Stokes flow case.

3.2 Small-Sample Stability and Physical Consistency

The proposed model remained stable when only a small number of sampled points were used for training. Under the 20% labeled-data setting, the model still gave reliable velocity, pressure, and temperature predictions. When the labeled ratio was reduced to 10%, the data-only neural network showed a clear increase in error. In contrast, the proposed model changed less. This result shows that PDE residuals can provide extra physical guidance when measured data are scarce. The basic PINN reduced error compared with the data-only model, but its residual error was still higher than that of the proposed model. This was because boundary and initial conditions were not fully included. Similar findings have been reported in heat-equation studies, where PINNs use boundary conditions and governing equations to reduce the need for dense training data [24,25]. Fig. 2 shows a related PINN heat-equation setting.

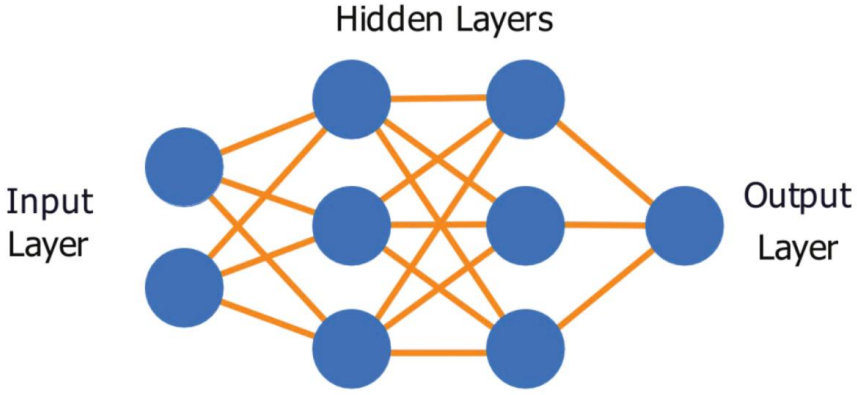


Fig. 2. **PINN structure for heat-equation modeling with boundary constraints.**

3.3 Generalization Under Changed Boundary Conditions

The boundary-transfer test showed that the proposed method reduced generalization error by about 21% when inlet velocity or wall temperature changed. The data-only neural network was more sensitive to boundary changes because it mainly learned patterns from the original samples. Once the boundary condition changed, its prediction error increased clearly. The basic PINN performed better because the PDE residual limited the solution space. However, local errors still appeared near the inlet, outlet, and wall regions. The proposed method further reduced these errors by adding boundary-condition loss and initial-condition loss. This result shows that physical constraints should include not only governing equations but also boundary and initial terms [26,27].

3.4 Comparison With Existing Studies and Engineering Value

Compared with data-driven models, the proposed method improved prediction accuracy and physical consistency. Compared with the basic PINN, it performed better under small samples and changed boundary conditions because more physical terms were included in the loss function. These results agree with recent PINN studies in fluid mechanics and heat transfer. Physical equations can reduce data demand and improve model reliability. However, the study also shows that PINNs are sensitive to loss weights and sampling quality. If the PDE loss is too weak, the model may lose physical consistency. If it is too strong, the model may fit the equation but deviate from measured data. Therefore, proper loss balance is important for engineering use. The method may be applied to flow prediction, thermal-field estimation, equipment monitoring, and fast engineering simulation. More tests are still needed in three-dimensional geometries, turbulent flow, coupled heat-flow systems, and noisy field measurements.

4.Conclusion

This study proposed a physics-constrained neural-network method for engineering modeling with limited data. The method added PDE residuals, boundary conditions, and initial conditions to the training loss. Thus, the model learned from both sampled data and physical equations. Tests on fluid-flow and heat-conduction cases showed that the method reduced prediction error by about 32% compared with a traditional neural network. Under changed boundary conditions, the generalization error decreased by about 21%. The model also remained stable with small training samples. These results show that physical constraints can improve prediction accuracy, reduce physically incorrect outputs, and improve model reliability. The main contribution of this study is the joint use of data fitting and physical constraints in one training process. This method may be used for flow prediction, thermal-field estimation, equipment monitoring, and fast engineering simulation. However, this study still has limitations. The tests were mainly based on two-dimensional cases and controlled boundary settings. More studies are needed for three-dimensional geometries, turbulent flow, coupled heat-flow systems, and noisy field data.

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